Name _____ Date _____

Algebra 2 Unit 10: Logarithmic Functions

Logarithmic Functions Pretest

1. Which of the following is equivalent to $y = \log_7 x$?

(1) $y = x^7$ (3) $x = 7^y$

(2) $x = y^7$ (4) $y = x^{1/7}$

2. If the graph of $y = 6^x$ is reflected across the line $y = x$ then the resulting curve has an equation of

(1) $y = -6^x$ (3) $x = \log_6 y$

(2) $y = \log_6 x$ (4) $x = y^6$

3. Which of the following represents the *y*-intercept of the function $y = \log(x + 1000) - 8$?

(1) -8 (3) 3

(2) -5 (4) 5

4. Determine the value for each of the following logarithms.

(a) $\log_2 32$ (b) $\log_7 49$ (c) $\log_3 6561$ (d) $\log_4 1024$

5. Determine the value for each of the following logarithms.

(a) $\log_2 \left(1/64\right)$ (b) $\log_3 (1)$ (c) $\log_5 \left(1/25\right)$ (d) $\log_7 \left(1/343\right)$

6. Determine the value for each of the following logarithms. Each of these will have non-integer, fractional answers.

(a) $\log_4 2$ (b) $\log_4 8$ (c) $\log_5 \sqrt[3]{5}$ (d) $\log_2 \sqrt[5]{4}$

7. Between what two consecutive integers must the value of $\log_4 7342$ lie? Justify your answer.

8. The domain of $y = \log_3(x+5)$ in the real number is

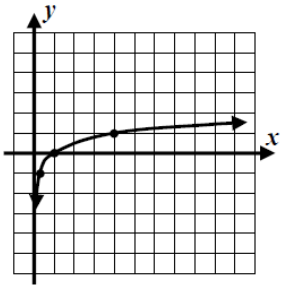
(1) $\{x \mid x > 0\}$ (3) $\{x \mid x > 5\}$

(2) $\{x \mid x > -5\}$ (4) $\{x \mid x \geq -4\}$

9. Which of the following equations describes the graph shown below?

(1) $y = \log_5 x$ (3) $y = \log_3 x$

(2) $y = \log_2 x$ (4) $y = \log_4 x$



10. Which of the following values of *x* is *not* in the domain of $f(x) = \log_5(10 - 2x)$?

(1) -3 (3) 5

(2) 0 (4) 4

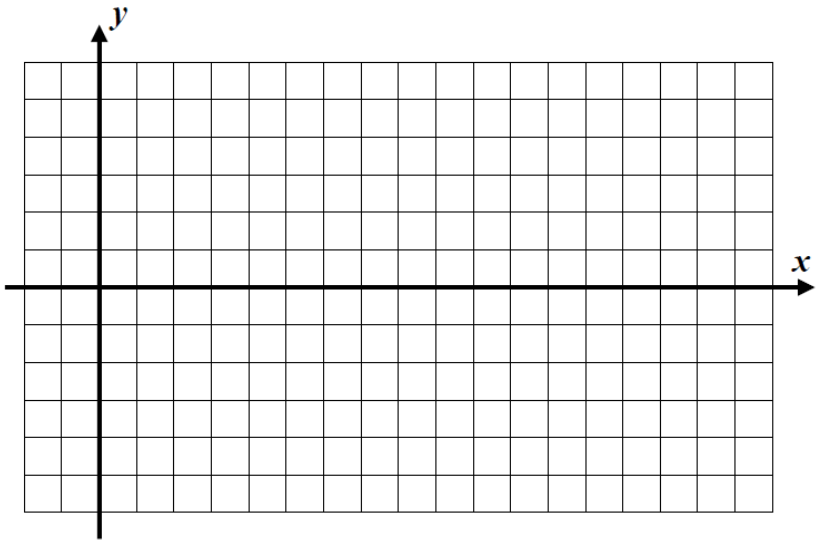
11. Consider the function $y = \log_4(x+16) - 1$. Identify the *x* and *y*-intercepts and the equation for the vertical asymptote.

12. Determine the domains of each of the following logarithmic functions. State your answers using any accepted notation. Be sure to show the inequality that you are solving to find the domain and the work you use to solve the inequality.

(a) $y = \log_5(2x-1)$

(b) $y = \log(6-x)$

13. Graph the logarithmic function $y = \log_4 x$ on the graph paper given. For a method, see *Example 1*. Be sure to plot points, label and scale your axes.



14. Which of the following is not equivalent to log 36?

(1) $\log 2 + \log 18$ (3) $\log 30 + \log 6$

(2) $2\log 6$ (4) $\log 4 + \log 9$

15. The $\log_3 20$ can be written as

(1) $2\log_3 2 + \log_3 5$ (3) $\log_3 15 + \log_3 5$

(2) $2\log_3 10$ (4) $2\log_3 4 + 3\log_3 4$

16. Which of the following is equivalent to $\log\left(x^3 \big/ \sqrt[3]{y}\right)$?

(1) $\log x - \log y$ (3) $3\log x - \frac{1}{3}\log y$

(2) $9\log(x - y)$ (4) $\log(3x) - \log\left(y \big/ 3\right)$

17. The difference $\log_2(3) - \log_2(12)$ is equal to

(1) -2 (3) $\frac{1}{4}$

(2) $-\frac{1}{2}$ (4) 4

18. If $\log 5 = p$ and $\log 2 = q$ then $\log 200$ can be written in terms of p and q as (Hint: write 200 as a product involving 5 and 2.)

(1) $4p + q$ (3) $2(p + q)$

(2) $2p + 3q$ (4) $3p + 2q$

19. When rounded to the nearest hundredth, $\log_3 7 = 1.77$. Which of the following represents the value of $\log_3 63$ to the nearest *hundredth*? Hint: write 63 as a product involving 7.

(1) 3.54
(3) 3.77
(2) 8.77
(4) 15.93

20. The expression $4\log x - \frac{1}{2}\log y + 3\log z$ can be rewritten equivalently as

(1) $\log\left(\frac{x^4 z^3}{\sqrt{y}}\right)$
(3) $\log\left(\frac{x^4 z^3}{2y}\right)$
(2) $\log\left(\frac{6xz}{y}\right)$
(4) $\log\left(\frac{6x^4 z^3}{y}\right)$

21. If $k = \log_2 3$ then $\log_2 48$ equals (Hint: write 48 as a product involving 3.)

(1) $2k + 3$
(3) $k + 8$
(2) $3k + 1$
(4) $k + 4$

22. If $g(x) = 8x^6$ and $f(x) = \log_4(2x)$ then $f(g(x)) = ?$ Show your work.

(1) $4\log_4 x + 1$
(3) $2(3\log_4 x + 1)$
(2) $3(\log_4 x + 2)$
(4) $6\log_4 x + 4$

23. Which of the following values, to the nearest *hundredth*, solves: $7^x = 500$.

(1) 3.19
(3) 2.74
(2) 3.83
(4) 2.17

24. The solution to $2^{x/3} = 52$, to the nearest *tenth*, is which of the following?

(1) 7.3 (3) 11.4

(2) 9.1 (4) 17.1

25. To the nearest *hundredth*, the value of x that solves $5^{x-4} = 275$ is

(1) 6.73 (3) 8.17

(2) 5.74 (4) 7.49

26. Solve each of the following exponential equations, **algebraically**. Round each of your answers to the nearest *hundredth*.

(a) $6^{2x-5} = 300$ (b) $\left(\frac{1}{2}\right)^{\frac{x}{3}+1} = \frac{1}{6}$ (c) $500(1.02)^{\frac{x}{12}} = 2300$

$\log (6)^{2x-5} = \log (300)$ $\log (½)^{x/3+1} = (\log 1/6)$ $\log (1.02)^{x/12} = \log (23/5)$

$2x\text{-}5(\log 6) = \log (300)$ $(x/3+1)(\log ½) = (\log 1/6)$ $(x/12)(\log 1.02) = \log (23/5)$

$2x - 5 = 3.183342$ $x/3 + 1 = 2.58496$ $x/12 = 77.063325$

$2x = 8.183342$ $x/3 = 1.58496$ $x = 924.76$

$x = 4.09$ $x = 4.75$

27. The population of Red Hook is growing at a rate of 3.5% per year. If its current population is 12,500, in how many years will the population exceed 20,000? Round your answer to the nearest year. Only an *algebraic* solution is acceptable.

$$\frac{12500(1.035)^x}{12500} = \frac{20000}{12500}$$

$$(1.035)^x = 8/5$$

$$\log (1.035)^x = \log (8/5)$$

$$x (\log 1.035) = \log (8/5)$$

$x = 13.66$ So, between years 13 and 14.

28. A radioactive substance is decaying such that 2% of its mass is lost every year. Originally there were 50 kilograms of the substance present.

(a) Write an equation for the amount, *A,* of the substance left after t-years.

$A = 50\ (.98)^t$

(b) Find the amount of time it takes for only half of the initial amount of remain. Round your answer to the nearest *tenth* of a year. Only an *algebraic* solution is acceptable

$50\ (.98)^t = 25$

$.98^{\ t} = ½$

$t \log (.98) = \log (½)$

$t = 34.3$ years

29. If a population doubles every 5 years, how many years will it take for the population to increase by 10 times its original amount? Round to the nearest tenth.

    First: If the population gets multiplied by 2 every 5 years, what does it get multiplied by each year? Use this to help you answer the question.

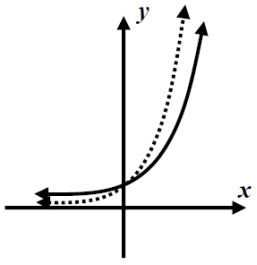
30. Find the solution to the general exponential equation $a(b)^{cx} = d$, in terms of the constants *a*, *c*, *d* and the logarithm of base *b*. Think about reversing the order of operations in order to solve for *x*.

31. Which of the following is closest to the *y*-intercept of the function whose equation is $y = 10e^{x+1}$?

    (1) 10
    (2) 18
    (3) 27
    (4) 52

32. On the grid below, the solid curve represents $y = e^x$. Which of the following exponential functions could describe the dashed curve? Explain your choice.

    (1) $y = \left(\frac{1}{2}\right)^x$
    (2) $y = e^{-x}$
    (3) $y = 2^x$
    (4) $y = 4^x$



33. The logarithmic expression $\ln\left(\frac{\sqrt{e}}{y^3}\right)$ can be rewritten as

(1) $3\ln y - 2$

(2) $\frac{1-6\ln y}{2}$

(3) $\frac{\ln y - 6}{2}$

(4) $\sqrt{\ln y} - 3$

34. Which of the following values of t solves the equation $5e^{2t} = 15$?

(1) $\frac{\ln 15}{10}$

(2) $\frac{1}{2\ln 5}$

(3) $2\ln 3$

(4) $\frac{\ln 3}{2}$

35. At which of the following values of x does $f(x) = 2e^{2x} - 32$ have a zero?

(1) $\ln\frac{5}{2}$

(2) $\ln 4$

(3) $\ln 8$

(4) $\ln\frac{2}{5}$

36. For the equation $ae^{ct} = d$, solve for the variable t in terms of a, c, and d. Express your answer in terms of the natural logarithm.

$ae^{ct} = d$

$e^{ct} = d/a$

$ct (\ln e) = \ln (d/a)$

$t = \frac{\ln (d/a)}{c}$

37. Flu is spreading exponentially at a school. The number of new flu patients can be modeled using the equation $F(d) = 10e^{0.12d}$, where *d* represents the number of days since 10 students had the flu.

(a) How many days will it take for the number of new flu patients to equal 50? Determine your answer algebraically using the natural logarithm. Round your answer to the nearest day.

(b) Find the average rate of change of *F* over the first three weeks, i.e. $0 \le d \le 21$. Show the calculation that leads to your answer. Give proper units and round your answer to the nearest tenth. What is the physical interpretation of your answer?

38. Solve the following equation for an **exact value of *t***.

$$12 = \frac{16e^{0.8t}}{e^{0.8t}+3}$$

39. The value of an initial investment of $400 at 3% nominal interest compounded quarterly can be modeled using which of the following equations, where t is the number of years since the investment was made?

(1) $A(t) = 400(1.0075)^{4t}$ (3) $A(t) = 400(1.03)^{4t}$

(2) $A(t) = 400(1.0075)^{t}$ (4) $A(t) = 400(1.0303)^{4t}$

40. Which of the following represents the value of an investment with a principal of $1500 with a nominal interest rate of 2.5% compounded monthly after 5 years?

(1) $1,697.11 (3) $4,178.22

(2) $1,699.50 (4) $5,168.71

41. Franco invests $4,500 in an account that earns a 3.8% nominal interest rate compounded continuously. If he withdraws the profit from the investment after 5 years, how much has he earned on his investment?

(1) $858.92 (3) $922.50

(2) $912.59 (4) $941.62

42. An investment that returns a nominal 4.2% yearly rate, but is compounded quarterly, has an effective yearly rate closest to

(1) 4.21% (3) 4.27%

(2) 4.24% (4) 4.32%

43. If an investment's value can be modeled with $A(t) = 325\left(1 + \frac{.027}{12}\right)^{12t}$ then which of the following describes the investment?

(1) The investment has a nominal rate of 27% compounded every 12 years.
(2) The investment has a nominal rate of 2.7% compounded ever 12 years.
(3) The investment has a nominal rate of 27% compounded 12 times per year.
(4) The investment has a nominal rate of 2.7% compounded 12 times per year.

44. An investment of $500 is made at 2.8% nominal interest compounded quarterly.

(a) Write an equation that models the amount *A* the investment is worth *t*-years after the principal has been invested.

(b) How much is the investment worth after 10 years? Show how you arrived at your answer.

(c) **Algebraically** determine the number of years it will take for the investment to reach a worth of $800. Round to the nearest *hundredth*.

45. A large city has a current population of 500,000 people which is decreasing continuously at a rate of 4.5% each year.

(a) Write an equation that models the population *P* after *t*, years.

(b) ***Algebraically***, determine the number of years it will take for the population to decrease to half of the current population. Round to the nearest hundredth.